A575 HW #5

Handed out: February 18, 2009
Due (at the beginning of class): February 25, 2009

1. [15 pts] Recall that the Salpeter IMF takes the functional form:

$$\xi(M) = \xi_0 M^{-2.35}$$

where ξ_0 sets the local stellar density.

(a) Integrate the Salpeter mass function between a lower mass limit M_l and upper limit $M_u >> M_l$ to (i) find the number of stars formed in a 100 pc cube, (ii) their total mass, and (iii) the total luminosity.

(b) Show that the number and mass of stars depend mainly on the mass M_l of the smallest stars, while the luminosity depends on M_u, the mass of the largest stars.

(c) Taking $M_l = 0.3$ $M_\odot$ and $M_u >> 5$ $M_\odot$, what fraction of all stars have $M > 5$ $M_\odot$? What fraction have $M > M_\odot$?

2. [15 pts] The metallicity of the gas increases as

$$\Delta Z = \Delta \left(\frac{M_h}{M_g} \right) = \frac{p\Delta M_s - Z[\Delta M_s + \Delta M_g]}{M_g}$$

where p is the yield, M_s is the mass of stars, M_g is the mass of gas, M_h is the mass of heavy elements, and Z is the original metallicity.

(a) Suppose that the inflow of gas is proportional to the mass of stars newly formed, so that $\Delta M_s + \Delta M_g = \nu \Delta M_s$ for some constant $\nu > 0$. Show that the above equation becomes:

$$\Delta Z = \frac{(p - \nu Z)\Delta M_s}{M_g} = \frac{(p - \nu Z)}{(\nu - 1)} \frac{\Delta M_g}{M_g}$$

(b) Derive the metallicity of the gas as a function of time.

(c) Discuss the implications of this inflow model on the maximum metallicity of the gas.

3. [20 pts] In order to calculate a gas-phase abundance from the observed emission line strengths, it is necessary to determine the physical conditions of the HII region. Observations of the several different transitions of the same species can be used as diagnostics. For the following problem, the relations between observed line strengths and temperature and density are approximate; a full treatment requires a multi-level atom.

Following Osterbrock (1989), p. 121, the ratio of the [O III] lines can be used as a temperature diagnostic:

$$\frac{j_{\lambda 4959} + j_{\lambda 5007}}{j_{\lambda 4363}} = \frac{7.73 \exp[(3.29 \times 10^4)/T_e]}{1 + 4.5 \times 10^{-4}(N_e/T_e^{1/2})}.$$

where j_λ is the emission coeficient of that transition, N_e is the electron density, and T_e is the electron temperature. Similarly, the ratio of the [S II] lines can be used as a density diagnostic:

$$\frac{j_{\lambda 6716}}{j_{\lambda 6731}} = 1.5 \times \frac{1 + 0.037859 N_e/T_e^{1/2}}{1 + 0.128139 N_e/T_e^{1/2}}$$

Since the forbidden lines are optically thin, we can assume that the ratio of the observed line intensities is the ratio of the emission coefficients.

The NW HII region of I Zw 18 has the following observed line strengths, relative to Hβ and corrected for reddening.

λ	I(ion)/I(Hβ)
[O II] 3727+3729	0.383 ± 0.017
[O III] 4363	0.059 ± 0.003
Hβ 4861	1.000 ± 0.035
[O III] 4959	0.655 ± 0.023
[O III] 5007	1.970 ± 0.069
Hα 6563	2.391 ± 0.113
[N II] 6584	0.009 ± 0.001
[S II] 6716	0.022 ± 0.001
[S II] 6731	0.015 ± 0.001

(a) Using the above relations, calculate the temperature and density of the ionized gas in this HII region.

(b) Using the FIVEL code (located in /home/vanzee/A575/fivel.f), compute the emission coefficients for the [O II] and [O III] and Hβ lines for this temperature and density combination.

(c) Calculate the abundance of O^+ and O^{++} relative to Hydrogen.

(d) Compute the oxygen abundance of I Zw 18. Express your answer both as O/H and 12+log(O/H).