Class notes for 9 November

Reminders
- Exercise 5 and Problem Set 5 due today
- Problem Set 6 due Nov. 16 (13.12, 13.13, 13.14, 14.3, 14.6)
- Remote observing and the Kitt Peak trip

14.1 Observations of Pulsating Stars

- o Ceti first found in 1595 by David Fabricius (Mira = wonderful)
- First "model" – brightness variations due to big starspots
- Really, it's a pulsating star, a long-period variable (about 11 month period)
- δ Cephei was next, 1784, 5.5 days – a classical Cepheid.
- 10's of thousands of variables now known, dozens of types of pulsating variables

- Period-luminosity relationship, Cepheids as standard candles
- Shapley proposed pulsation
- Eddington provided the theoretical framework

- Temperature varies by 1000-1500K
- Radius also changes, but lesser factor in amplitude
- **Phase lag**: brightest when expanding outward most rapidly (after minimum radius)

- Variables located in the instability strip in the HR diagram.
- Stars evolve into and out of regions of instability.

### Types of variables

<table>
<thead>
<tr>
<th>Type</th>
<th>Range of Periods</th>
<th>Population Type</th>
<th>Radial or Non Radial</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long Period Var.</td>
<td>100-700 days</td>
<td>I, II</td>
<td>R</td>
</tr>
<tr>
<td>Classical Cepheids</td>
<td>1-50 days</td>
<td>I</td>
<td>R</td>
</tr>
<tr>
<td>W Vir Stars</td>
<td>2-45 days</td>
<td>II</td>
<td>R</td>
</tr>
<tr>
<td>RR Lyrae Stars</td>
<td>1.5-24 hours</td>
<td>II</td>
<td>R</td>
</tr>
<tr>
<td>δ Scuti Stars</td>
<td>1.3 hours</td>
<td>I</td>
<td>R, NR</td>
</tr>
<tr>
<td>β Cephei Stars</td>
<td>3-7 hours</td>
<td>I</td>
<td>R, NR</td>
</tr>
<tr>
<td>ZZ Ceti Stars</td>
<td>100-1000 sec</td>
<td>I</td>
<td>NR</td>
</tr>
</tbody>
</table>

14.2 Physics of Stellar Pulsation

Stellar pulsations are the result of sound waves resonating in a star's interior (standing waves)

Estimate the pulsation period: how long for a sound wave to cross the diameter of a star (assume a radius R and a constant density ρ).  

**Estimate the pulsation period:**

\[ T = \frac{2R}{c} \]

where:
- \( T \) is the period of pulsation
- \( R \) is the radius of the star
- \( c \) is the speed of sound in the star

**Example:**

If \( R = 10^6 \text{ km} \) and \( c = 10^5 \text{ km/s} \), then

\[ T = \frac{2 \times 10^6}{10^5} = 20 \text{ seconds} \]
The adiabatic sound speed (Eqn 10.76) is:

\[ v_s = \sqrt{\frac{\gamma p}{\rho}} \]

With the assumption of uniform density, the pressure is:

\[
\frac{dP}{dr} = -\frac{GM_r\rho}{r^2} = -\frac{G(\frac{4}{3} \pi r^3 \rho)\rho}{r^2} = -\frac{4}{3} \pi G \rho^2 r
\]

Since \( P=0 \) at the surface, integrating this equation gives

\[ P(r) = \frac{2}{3} \pi G \rho^2 (R^2 - r^2) \]

So, the period is roughly (the period-density relation):

\[ \Pi \approx \frac{3\pi}{2\gamma G \rho} \]

Why does period decrease as you move down the instability strip from Cepheids to \( \delta \) Scuti stars?

The sound waves can resonate in the

- fundamental mode – node at the center
- first overtone mode – one node between the center and the surface
- second overtone mode – two modes between the center and the surface

Most pulsate in the fundamental mode.

What powers these standing waves? Eddington's model

**Thermodynamic heat engines**

- The layers do PdV work as they expand and contract.
- If the integrated energy PdV over a cycle is positive, the oscillations grow
- If it isn't, they damp out.

Heat enters a layer when it is hot, leaves a layer when it is cooler.

Driving mechanism NOT nuclear reactions: displacement very small at the center of a star, not enough to drive a pulsation.
**Eddington's valve mechanism**: if a layer becomes more opaque when it is compressed, it will absorb energy. It heats and expands, and pushes the upper layers outward. As it expands it cools, and the opacity drops, and the layer falls back down again.

In most of the star, the gas doesn’t behave this way. The opacity decreases when the gas is compressed and heats up.

But opacity does increase with compression in partial ionization zones (regions where hydrogen or helium is partially ionized). When the gas is compressed, the amount of ionization decreases and the opacity increases. Known as the κ-mechanism.

Two main ionization zones:
- **Hydrogen partial ionization zone** (H I > H II and He I > He II) at temperatures of 10,000 – 15,000 K
- **He II partial ionization zone** at temperatures around 40,000 K

If a star has a temperature above about 7500 K, these zones are too near the surface, and there isn't enough mass to drive a pulsation.

If the layers are deeper, the fundamental or first overtone modes may be excited.

If the temperature is less than about 5500 K, the convection zone damps out oscillations (too much heat is transported by convection to drive the pulsation).

The He II ionization zone is primarily responsible for pulsations in the instability strip (for LPVs, it’s the hydrogen ionization zone).