Class notes for 5 October

Reminders

• Problem Set 3 due Oct. 7 (9.7, 9.11, 9.27, 10.9, 10.13, 10.15)
• Exercise 3 due date postponed until Oct. 7
• Patten Lectures by Geoff Marcy, Oct. 12 and 14, 7:30 Rawles 100
• Physics/Astronomy Colloquium, Oct. 13, 4 PM, SW119
• Class with Geoff Marcy on Oct. 14 in SW 113

10.3 Stellar Energy Sources

Big ideas

• Gravitational and chemical energy are insufficient for the age of the Sun
• Energy from nuclear fusion does provide enough energy
• Proton-proton chain
• CNO cycle
• Triple-alpha process

Kelvin-Helmholtz time scale - about $10^7$ years, the time scale that it would take the Sun to radiate away the thermal energy produced from its gravitational contraction to its present size.

Definitions:

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<tr>
<th>Element</th>
<th>Coulomb barrier</th>
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<tr>
<td>Isotope</td>
<td>Quantum mechanical tunneling</td>
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<td>Nucleon</td>
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<td>Strong nuclear force</td>
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<td>Reduced mass</td>
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Proton-Proton Chain (PP I) - process for conversion of hydrogen into helium:

\[ _1^4 H \rightarrow _2^4 He + 2e^+ + 2\nu_e + 2\gamma \]

\[ _1^1 H + _1^1 H \rightarrow _1^2 H + \_ + \_ \]

\[ _1^2 H + _1^1 H \rightarrow \_ + \_ \]
\[ \frac{3}{2} He + \frac{3}{2} He \rightarrow \_ + \_ \]

About 70% of the time, two \(^3\text{He}\) nuclei interact to complete the PP I chain, but about 30% of the time, \(^3\text{He}\) and \(^4\text{He}\) interact (PP II):

\[ \frac{3}{2} \text{He} + \frac{4}{2} \text{He} \rightarrow \frac{7}{4} \text{Be} + \gamma \]

\[ \frac{7}{4} \text{Be} + e^- = \_ + \_ \]

\[ \frac{7}{3} \text{Li} + \frac{1}{1} \text{H} \rightarrow \]

And sometimes the \(^7\text{Be}\) nucleus captures a proton instead (PP III):

\[ \frac{7}{4} \text{Be} + \frac{1}{1} \text{H} = \frac{8}{5} \text{B} + \gamma \]

\[ \frac{8}{5} \text{B} \rightarrow \frac{8}{4} \text{Be} + e^+ + \nu_e \]

\[ \frac{8}{4} \text{Be} \rightarrow 2 \_ \]

**CNO Cycle**: Carbon, nitrogen, and oxygen are catalysts for the conversion of hydrogen into helium. Again, there are multiple branches. The basic steps are simple. Add a proton. If the resulting nucleus is stable (\(^{13}\text{C}, \text{^{14}N}, \text{^{15}N}\)), add another one. If the resulting nucleus is unstable (\(^{13}\text{N}, \text{^{15}O}\)), let it beta decay.

The rate of the CNO cycle depends on a high power of the temperature \(T_6^{19.9}\), compared to the PP cycle, at about \(T_6^{4}\).

Massive stars, with high central temperatures, are dominated by the CNO cycle. Low mass stars, with lower central temperatures, are dominated by the PP cycle.

As hydrogen burning proceeds, the mean molecular weight of the gas increases. To compensate for the increase, the central temperature and density must increase.
**Triple-alpha Cycle**

\[ ^4He + ^4He \rightarrow ^8Be \quad ^8Be + ^4He \rightarrow ^{12}C + \gamma \]

\(^8\text{Be}\) is highly unstable, and will decay immediately if another \(^{4}\text{He}\) nucleus is not captured.

The triple-alpha process depends on temperature to an even higher power \((T_8^{41})\). Increasing the temperature only 10% increases the energy output rate by more than a factor of 50.

**Nucleosynthesis, binding energy per nucleon** \(^{56}\text{Fe}\).

### 10.4 Energy Transport and Thermodynamics

- **Radiation** – energy carried via photons
- **Convection** – energy carried by buoyant, hot blobs of gas
- **Conduction** – energy transport via collisions between atoms and nuclei, mostly unimportant

Temperature gradient for **radiative transport**:

\[
\frac{dT}{dr} = -\frac{3}{4ac} \frac{\kappa}{T^3} \frac{L_r}{4\pi r^2} \quad (\text{a is the radiation constant})
\]

The temperature gradient increases as either luminosity or opacity increase.

If the temperature gradient becomes too steep, the gas becomes unstable to convection, and convection begins to dominate the energy transport. A detailed model of convection is hard (fluid mechanics, 3d, turbulence, comparable in scale length to the star and to the time scale for structural changes in the star – not a steady state condition).

- **Pressure scale height**
- **Specific heat**
- **Specific volume**
- **Adiabatic process**
- **Bulk modulus**

**Adiabatic temperature gradient**: temperature gradient at which a bubble of hot gas rises without exchanging heat with its surroundings (the bubble expands as it rises into a lower pressure region, and cools).

If the temperature gradient is steeper than adiabatic, convection will occur. Generally, convection will occur when
opacity is high (radiation cannot transport heat outward easily if the opacity is high, and this leads to a high temperature gradient).
- ionization is occurring (large specific heat and a low adiabatic temperature gradient)
- gravity is low, leading to a low adiabatic gradient
- energy generation is a very steep function of temperature (steep flux gradient and a steep temperature gradient)

**Mixing length theory**: a 1-d, phenomenological description of convection. Still, it's pretty successful. The theory is based on the ratio of the "mixing length" to the pressure scale height. The mixing length is defined as the distance a hot bubble of gas travels before thermalizing with its surroundings.