Class notes for 2 September

Reminders
- First problem set due Sept. 7 (problems 3.8, 3.13, 5.1)
- First class exercise due Sept. 9
- First take-home exam handed out Sept. 9, due at beginning of class Sept. 14

3.3 The Wave Nature of Light

Speed of light: \(3 \times 10^{10} \text{ cm/sec} \quad c = \lambda \nu\)

<table>
<thead>
<tr>
<th>Spectral Region</th>
<th>Wavelength (Angstroms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gamma Ray</td>
<td>(\lambda &lt; 0.1 \text{ A})</td>
</tr>
<tr>
<td>X-ray</td>
<td>(0.1 \text{ A} &lt; \lambda &lt; 100 \text{ A})</td>
</tr>
<tr>
<td>Ultraviolet</td>
<td>(100 \text{ A} &lt; \lambda &lt; 4000 \text{ A})</td>
</tr>
<tr>
<td>Visible</td>
<td>(4000 \text{ A} &lt; \lambda &lt; 7000 \text{ A})</td>
</tr>
<tr>
<td>Infrared</td>
<td>(7000 \text{ A} &lt; \lambda &lt; 1 \text{ mm})</td>
</tr>
<tr>
<td>Microwave</td>
<td>(1 \text{ m} &lt; \lambda &lt; 10 \text{ cm})</td>
</tr>
<tr>
<td>Radio</td>
<td>(10 \text{ cm} &lt; \lambda)</td>
</tr>
</tbody>
</table>

3.4 Blackbody Radiation

Wien’s Displacement law: \(\lambda_{\text{max}} T = 0.290 \text{ cm K}\)

Example 3.4: Betelgeuse has a surface temperature of 3400 K. If we treat Betelgeuse as a blackbody, Wien’s displacement law shows that its continuous spectrum peaks at a wavelength of

\[\lambda_{\text{max}} = \frac{0.290 \text{ cm K}}{3400 \text{ K}} = 8.53 \times 10^{-5} \text{ cm} = 8530 \text{ A}\]

which is in the infrared region of the electromagnetic spectrum. Rigel, with a surface temperature of 10,100 K, has a continuous spectrum that peaks at a wavelength of

\[\lambda_{\text{max}} = \frac{0.290 \text{ cm K}}{10100 \text{ K}} = 2.87 \times 10^{-5} \text{ cm} = 2870 \text{ A}\]

in the ultraviolet.

Stefan-Boltzmann equation: \(L = 4\pi r^2 \sigma T^4 \quad (\sigma = 5.670 \times 10^{-5} \text{ erg/sec/cm}^2/\text{K}^4)\)

This equation defines the effective temperature of a star.
**Example 3.5** The luminosity of the Sun is $L = 3.826 \times 10^{33}$ ergs/sec and its radius is $R = 6.69 \times 10^{10}$ cm. The effective temperature of the Sun's surface is then

$$T_{\text{Sun}} = \left( \frac{L_{\text{Sun}}}{4\pi R^2 \sigma} \right)^{\frac{1}{4}} = 5770 \text{K}$$

The radiant flux at the solar surface is $F_{\text{surf}} = \sigma T^4 = 6.285 \times 10^{10}$ erg/sec/cm$^2$.

**Definitions** (figure to be added later - scanner not working!)

- $dA$ - Unit surface area
- $\phi$, $d\phi$ - "azimuthal" angle
- $\theta$, $d\theta$ - "elevation" angle
- $\Omega$, $d\Omega$ - solid angle = $\sin \theta d\theta d\phi$
- $r$ is the distance to the star
- $R$ is the radius of the star

The Planck function connects the observed properties of a star (radiant flux, apparent magnitude) to its intrinsic properties (radius, temperature). The Planck function gives the amount of energy per unit wavelength (or frequency) interval emitted by a black body of surface area $dA$ into a solid angle $d\Omega = \sin \theta d\theta d\phi$.

$$B_\lambda (T) = \frac{2hc^2}{e^{hv/kT} - 1} \quad B_\nu (T) = \frac{2h\nu^3/c^2}{e^{h\nu/kT} - 1}$$

$h = 6.626 \times 10^{-27}$ erg sec

The monochromatic luminosity of a star is the integral of the Planck function over angles $0<\phi<2\pi$ and $0<\theta<\pi/2$ and over the surface area of the star.

$$L_\lambda d\lambda = 4\pi^2 R^2 B_\lambda d\lambda = \frac{8\pi^2 R^2hc^2/\lambda^5}{e^{hc/\lambda kT} - 1}$$

Integrating $L_\lambda d\lambda$ over all wavelengths gives the total luminosity, which we know equals $4\pi^2\sigma T^4$.

The monochromatic flux is the number of ergs per second with wavelength between $\lambda$ and $\lambda+d\lambda$ at a detector with area one square centimeter aimed at the star.
\[
F_\lambda d\lambda = \frac{L_\lambda}{4\pi r^2} d\lambda = \frac{2\pi hc^2}{e^{hc/\lambda kT} - 1} \left(\frac{R}{r}\right)^2 d\lambda
\]

### 3.6 Color Index

Apparent magnitudes are measured through specific filters, often the UBV filters
- **U** (ultraviolet) centered at 3650A, 680A bandwidth
- **B** (blue) centered at 4400A, 980A bandwidth
- **V** (visual) centered at 5500A, 890A bandwidth

Magnitudes are defined in each filter. Differences in magnitude between two filters are called **color indices**. (Why?)

The bolometric correction is the difference between a star's apparent bolometric magnitude and its apparent visual magnitude, or between its absolute bolometric magnitude and its absolute visual magnitude:

\[
BC = m_{bol} - V = M_{bol} - M_V
\]

**Note that bolometric corrections are almost always NEGATIVE, since brighter objects have smaller magnitudes.**

**Example 3.6** *Sirius, the brightest appearing star in the sky, has U, B, and V apparent magnitudes of U=-1.50, B=-1.46, and V=-1.46. Thus, for Sirius,*

\[
\begin{align*}
U-B & = -1.50 - (-1.46) = -0.04 \\
B-V & = -1.46 - (-1.46) = 0.00
\end{align*}
\]

*Sirius is brightest at ultraviolet wavelengths, as expected for a star with an effective temperature of Te=9910K. For this surface temperature,*

\[
\lambda_{max} = \frac{(5000A)(5800A)}{9910K} = 2930A
\]

*which is in the ultraviolet portion of the electromagnetic spectrum. The bolometric correction for Sirius is* \(BC = -0.09\), so its apparent bolometric magnitude is

\[
m_{bol} = V + BC = -1.46 + (-0.09) = -1.55
\]