Class notes for 21 September

Reminders
• Exercise 2 due NOW
• Problem set 2 NOW
• Remote observing Sept. 26, 27

9.3 Radiative Transfer

The flow of radiation through a gas involves not just absorption, but also emission. Emission includes both true emission (from electrons making downward transitions) and scattering. Absorption and emission redirect the path of a photon and redistribute energy. Photons diffuse outward through "random walk" that doesn't transport energy very efficiently.

"Optional" part (NOT COMPLETELY OPTIONAL)

Consider the intensity of a ray of light passing through a gas. Emission processes add to the intensity of the light, and absorption processes reduce the intensity.

Recall that $dI_\lambda = \kappa_\lambda \rho ds$ describes the loss of intensity from absorption, where $\kappa_\lambda$ is the absorption coefficient per gram.

Likewise, the ray gains intensity from emission processes: $dI_\lambda = j_\lambda \rho ds$, where $j_\lambda$ is the emission coefficient, with units of cm s$^{-3}$ sr$^{-1}$.

Combining these two gives the total change of intensity with distance:

$$dI_\lambda = \kappa_\lambda \rho ds + j_\lambda \rho ds$$

Dividing by $-\kappa_\lambda \rho ds$ gives

$$-\frac{1}{\kappa_\lambda \rho} \frac{dI_\lambda}{ds} = I_\lambda - \frac{j_\lambda}{\kappa_\lambda}$$

The ratio of the emission coefficient divided by the absorption coefficient is call the source function, $S_\lambda = j_\lambda/\kappa_\lambda$. The source function has units of erg s$^{-1}$ cm$^{-3}$ sr$^{-1}$.

We write the equation of radiative transfer as

$$-\frac{1}{\kappa_\lambda \rho} \frac{dI_\lambda}{ds} = I_\lambda - S_\lambda$$

If the intensity of light does not vary, then the intensity is equal to the source function ($S_\lambda = I_\lambda$).

When we assume local thermodynamic equilibrium, we are making the assumption
\( S_\lambda = B_\lambda \)

If we assume that every energy process is in equilibrium with its reverse process (as in a blackbody, then set \( I_\lambda = B_\lambda \), and \( j_\lambda/\kappa_\lambda = B_\lambda \).

Example 9.5

The equation of radiative transfer can be written in terms of optical depth

\[
\frac{dI_\perp}{dT_\perp} = I_\perp - S_\perp
\]

Simplifying Assumptions

- Consider a "not-spherical" star - the depth of a star's atmosphere is thin compared to the star's radius - assume the atmosphere is plane parallel.

- Assume that opacity is independent of wavelength - gray atmosphere

  - Then \( I = \int I_\lambda d\lambda \) and \( S = \int S_\lambda d\lambda \)

- If the star is in thermodynamic equilibrium (every process of absorption is balanced by a process of emission) no net energy is added or subtracted from the radiation. This means that the total flux is constant with depth,

  \( F_{\text{rad}} = \text{constant} = F_{\text{surf}} = \sigma T_e^4 \).

- Eddington Approximation - Assume the intensity of the radiation has one value (\( I_{\text{out}} \)) in all directions of the outward facing hemisphere, and another value (\( I_{\text{in}} \)) in all directions of the inward facing hemisphere.

These assumptions lead to a physical description of a gray atmosphere

\[
T^4 = \frac{3}{4} T_e^4 \left( \tau_v + \frac{2}{3} \right)
\]

So what...

\( T = T_e \) at \( \tau_v = 2/3 \), not at \( \tau_v = 0 \), so the "surface" of a star is actually at optical depth 2/3

Compute a gray atmosphere for the Sun: Assuming that the effective temperature of the Sun is 5770K, compute a \( T(\tau) \) relation for the Sun using a gray atmosphere and the Eddington approximation. Determine \( T \) at optical depths \( \tau = 0, 1/2, 2/3, 1, \) and 2.
The top graph shows the comparison between Holweger Muller Atmosphere and Gray Atmosphere as a function of Optical Depth (Tau Rosseland) and Temperature (K).

The bottom graph illustrates the intensity (I(theta)/I(0)) as a function of Cos theta, with Eddington Approx. and Observed Intensity plotted.

The data points and curves represent the theoretical and observed behaviors in these atmospheric models.