Astronomy 451 – Stellar Astrophysics

Solutions for Exam 4

1. An interstellar cloud with a mass of 300 solar masses has a temperature of 75 K and a particle density n = 10^5 cm^{-3}. Assume the cloud is composed entirely of H I, so that the mean molecular weight μ = 1. Is the cloud stable against gravitational collapse?

An interstellar cloud is unstable against gravitational collapse if its mass is greater than the Jeans mass, where

$$M_J \cong \left(\frac{5kT}{G\mu m_H}\right)^{3/2}\left(\frac{3}{4\pi\rho_0}\right)^{1/2}$$

Since the cloud is composed of H I, then $\rho_0 = 1.67x10^{-19}$ *g* cm^{-3}.

The Jeans mass for this cloud would be

$$M_J \cong \left(\frac{5x1.38x10^{-16}x75}{6.67x10^{-8}x1.0x1.67x10^{-24}}\right)^{3/2}\left(\frac{3}{4x3.14x1.67x10^{-19}}\right)^{1/2} = \left(\frac{517.5x10^{-16}}{11.1x10^{-32}}\right)^{3/2}\left(\frac{3}{21x10^{-19}}\right)^{1/2} = 3.8x10^{35}\, grams$$

Since one solar mass is $2x10^{33}$ *grams, this corresponds to 190 solar masses. Clouds of this density with masses larger than 190 solar masses will collapse due to gravity. Smaller clouds will be stable.*

This cloud is unstable to gravitational collapse.

Many students compared the thermal and potential energies, and this also works.

2. An often used formula for estimating the mass loss rate of stars was developed by Reimer, and is shown in question 13.5 of the text. Integrating the Reimer's mass loss formula gives an expression for the mass of a star as a function of time:

$$M = (M_0^2 - 8x10^{-13}LRt)^{1/2}$$

R, L, and M are in solar units, and t is in years. For a star which began with 3 solar masses before evolving to the red giant phase, estimate the time necessary *on the red giant branch* to lose the hydrogen envelope. Is this time consistent with the red giant lifetime of such a star? Figure 13.1 and Table 13.1 will prove useful.

Assume the star arrives at the giant branch with a mass of three solar masses. From the evolutionary tracks in Figure 13.1, a 3 solar mass giant star will have a radius of about 15 times the solar radius, and its luminosity will be about 100 times the luminosity of the Sun. To lose all of its envelope, it will need to lose about 2+ solar masses of material. Thus,

$1 = (3^2 - 8x10^{-13}x100x15t)^{1/2}$ *or* $t = 7 \times 10^9$ *years.*

This is much longer than the giant branch lifetime of a 3 solar mass star, as given in Figure 13.1 and Table 13.1.

3. A Cepheid variable with a mean apparent magnitude of V=10 and a period of 22 days is observed in a galactic cluster. What is the distance to the cluster? Assume there is no reddening.

From Figure 14.4 (or equation 14.2), a Cepheid with a period of 22 days (log P = 1.34) has an absolute visual magnitude of about -5. The distance modulus is then 15 magnitudes.

$d = 10^{(m-M+5)/5}$ *parsecs, or d is about 10 kpc.*

4. Estimate the number of white dwarfs in a volume 100 parsecs on a side near the Sun. Describe your method clearly and state any assumptions.

Several methods can be used for this estimate.

Method 1: From Figure 15.10, one can estimate that there are perhaps a few times 10^{-3} *white dwarfs per cubic parsec. A cube 100 parsecs on a side has a volume of a million cubic parsecs. There would be a few thousand white dwarfs in this volume.*

Method 2: The nearest white dwarf to the Sun is a couple of parsecs away. From this one can crudely estimate that there must be fewer than 1 white dwarf per cubic parsec, and probably fewer than 0.1 per cubic parsec. If the number is as high as 0.1 per cubic parsec, there would be 100,000 white dwarfs in this volume.

Method 3: You know that about 25% of stars are white dwarfs. You can crudely estimate the local space density of stars at about 1 per cubic parsec (actually, it's less!), so you could estimate that there are 0.25 white dwarfs per cubic parsec. This would lead to an estimate of 250,000 in a 100-parsec cube.

The actual local space density of white dwarfs is about $5.5x10^{-3}$ *per cubic parsec, so one should expect about 5500 white dwarfs in a 100-parsec cube.*