Astronomy 451 – Stellar Astrophysics

Exam 3 Solutions, Chapters 10 & 11

1. Estimate the pressure at the center of Jupiter from the equation of hydrostatic equilibrium, assuming the average jovian density is 1.3 g cm^{-3}. Jupiter's mass and radius are 1.9 x 10^{30} g and 7.2 x 10^{9} cm, respectively. How does the density at the center of Jupiter compare with the density at the center of the Sun? The average density of the Sun is about 1.4 g cm^{-3}. State your assumptions and show your work. (20 points)

$$\frac{dP}{dr} = \frac{P_s - P_c}{r_s - r_c} = -\frac{P_c}{r_{jup}} = -G\frac{M_{jup}\rho}{r_{jup}^2} = -\rho g$$

P_c = 2.3 x 10^{13} dynes/cm^2

There is not enough information to compare the densities, except through the ideal gas law dependence on mean molecular weight. You can conclude that the densities will differ since the Sun's core is ionized gas while Jupiter's core is not.

2. Estimate the temperature at the core of Jupiter, assuming the ideal gas law applies. You may assume that Jupiter's composition is pure hydrogen for the purpose of this calculation. (20 points)

$$T_c = \frac{P_c \mu m_H}{\rho k}$$

P_c = 2.3 x 10^{13} dynes/cm^2
m_H = 1.7 x 10^{-24} grams
k = 1.4 x 10^{-16} erg/K
and assume ρ = 1.3 gm cm^{-3}

The value of the mean molecular weight depends on whether the hydrogen is ionized. If not, μ = 1; if so μ = 0.5. Calculate T_c assuming one or the other, and then decide. Or, you could remember that Jupiter might have a metallic hydrogen core, which would not be ionized….

Thus, T_c comes out to about 200,000 from this simple calculation. Actually, the core temperature is more like 25,000, because the assumption of the ideal gas law isn't valid.

3. If a 0.5 solar mass star with a radius of 0.6 solar radii were to suddenly shrink in radius by 1%, how much energy would be liberated? If the initial luminosity of the star is 6% of the solar luminosity, how would the amount of energy liberated compare to its luminosity? (20 points)

$$E \approx \frac{3GM^2}{10R}$$

M = 0.5 x 2 x 10^{33} *gm =* 10^{33} *gm*
R = 0.6 x 7 x 10^{10} *cm = 4.2 x* 10^{10} *cm initially, and it shrinks to 4.16 x* 10^{10} *cm*

The difference in energy at these two radii is 4.8 x 10^{45} *ergs.*

The luminosity of the star is 0.06 x 4 x 10^{33} *erg* s^{-1} *= 2.4 x* 10^{32} *erg* s^{-1} *, so the energy released by the sudden contraction is 2 x* 10^{13} *greater.*

4. Estimate the amount of energy that will be produced in the Sun during its carbon-burning phase (m(^{4}He) = 6.6465 x 10^{-24} g, m(^{12}C) = 19.9265 x 10^{-24} g). You may assume that 0.5 solar masses of helium will be converted to carbon. How does the amount of energy produced by helium burning compare to the total energy produced by hydrogen burning during the Sun's main sequence lifetime (assume a main sequence lifetime of 10 billion years)? (20 points)

0.5 solar masses is 10^{33} *grams, which contains 1.5 x* 10^{56} *helium atoms. These are converted to 5 x* 10^{55} *carbon atoms. For each conversion, 0.013 x* 10^{-24} *grams are converted into energy.*

The total mass converted into energy is 5 x 10^{55} *x 0.013 x* 10^{-24} *grams = 6.5 x* 10^{29} *grams.*

Since $E=mc^2$*, the energy produced is 6.5 x* 10^{29} *x 9 x* 10^{20} *ergs = 5.9 x* 10^{50} *ergs.*

The main sequence lifetime of the Sun is about 10^{10} *years, and the luminosity of the sun is 4 x* 10^{33} *ergs* s^{-1}*. During its main sequence lifetime, the sun produces 3 x* 10^{7} *s* $year^{-1}$ *x* 10^{10} *years x 4 x* 10^{33} *ergs* s^{-1} *= 1.2 x* 10^{51} *ergs.*

The helium burning phase produces about half the energy produced during the main sequence lifetime of the Sun.

5. Considering that the Sun's corona has a temperature of 2 x 10^{6} K, why doesn't the Sun appear as a blackbody at this temperature? (5 points)

The density of the gas is very low and the flux from the corona is much fainter than the flux from the stellar photosphere.

6. What nuclear reaction process is likely to dominant in the core of:(5 points)

a main sequence B0 star? _____*CNO Cycle*______________________________

the Sun? ______________*PP Cycle*_____________________________________

a 0.2 solar mass M dwarf? _____*PP Cycle*_____________

7. Describe the origin of the solar wind, including its relationship to structures and mechanisms of the Sun's outer atmosphere. (5 points)

- *The corona has a very high temperature, and the matter is a plasma.*
- *Plasmas conduct heat well, so the corona is almost isothermal.*
- *Because of the high temperature, the gas pressure is higher than permitted by hydrostatic equilibrium and it is not in hydrostatic equilibrium.*
- *The plasma of the corona is expanding into space.*
- *Because the gas is a plasma, the ions are constrained to follow magnetic field lines.*
- *The wind moves outward where the magnetic field lines are open (coronal holes).*

8. Describe the origin of the sunspot cycle, including how the basic observations of the behavior of sunspots can be accounted for in the model. (5 points)

- *The magnetic field exists predominantly in the convection zone.*
- *Differential rotation drags and stretches the magnetic field lines, convection twists them.*
- *Regions of higher intensity magnetic field are buoyant and rise through the photosphere, to appear as sunspots.*
- *The polarity of the sunspots is due to the direction of the magnetic field (opposite in opposite hemispheres).*
- *Twisting occurs first at higher latitudes, so sunspots appear their first (butterfly diagram).*
- *As the field becomes more twisted, sunspots appear at lower latitude, and more sunspots appear.*
- *Fields cancel near the equator, and the poloidal field reestablishes, with reversed polarity. The number of sunspots is at a minimum*